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Functional Analysis for Gauge Fields on the Front-Form and the Light-Cone Gauge

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Abstract Constrained systems in quantum field theories call for a careful study of diverse classes of constraints and consistency checks over their temporal evolution. Here we study the functional structure of the free electromagnetic and pure Yang–Mills fields on the front-form coordinates with the null-plane gauge condition. It is seen that in this framework, we can deal with *strictu sensu* physical fields.

1 Introduction

In 1949 Dirac [1] showed in his pioneering work entitled *"Forms of Relativistic Dynamics"* that different choices of the time evolution parameter can drastically change the content and interpretation of the theory. The simplest and best-known example that illustrates this observation is the front-form dynamics of the free scalar field model [2] in contrast to the instant-form. In such a case, due to the choice of the time evolution parameter along the front-form [3–5], the theory becomes degenerate, and thus, the corresponding Hessian vanishes [6–8].

A given dynamical physical theory in the front-form becomes severely constrained with many second-class constraints. These can be eliminated by constructing Dirac brackets, making it possible to be quantized canonically by the correspondence principle in terms of a reduced number of independent fields [9]. The commutation relations among the field operators are found by Dirac's method, and they are used to obtain the momentum space expansion of the fields [10].

Since the front-form coordinates are not related to the conventional coordinates by a Lorenz transformation, the description of the same physical results may be different in the equal time and equal front-form time formulations of the theory. This was found to be the case in a study of some soluble two-dimensional

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gauge theory models [11,12], where it was demonstrated that the front-form quantization is very economical in displaying the relevant degrees of freedom, leading directly to a physical Hilbert space.

The main advantage of the front-form quantization is the apparent simplicity of the vacuum state. Indeed, naïve kinematical arguments suggest that the physical vacuum is trivial on the front-form [13]. If the discretization method is used as an infrared regulator [14], the dynamical vacuum structure will be connected to the $k^+ = 0$ Fourier modes of the fields. Studies of model field theories have shown that certain aspects of vacuum physics can in fact be reproduced by a careful treatment of the field zero modes. For example, the zero mode constraint equation in ϕ^4_{1+1} [15] exhibits spontaneous symmetry breaking [16].

The front-form formulation of quantum gauge field theories has been popular in the last few years; yet, the original attempts at setting up canonical quantization for QED in such framework and in the light-cone gauge $A_- = 0$, has been known for almost forty years [3–5]. This original formulation involved only physical degrees of freedom and, at the perturbative level. It was realized that the light-cone was appropriate for the computation of quantum effects contributing to the leading logarithm approximation in deep-inelastic processes [17,18]. However, some problems were associated with such gauge choice: Feynman amplitudes at the one-loop level exhibited double-pole singularities [18]. This pathological behavior has been ascribed to the Principal Value (PV) prescription employed to treat the unphysical poles $(k \cdot n)^{-1}$ of the gauge boson propagator [19], though application of dimensional regularization to one-loop integrals does also yield double poles [18,20]: These double poles appeared to be prescription independent.

Mandelstam and Leibbrandt [21,22] independently authored two prescriptions to handle the $(k \cdot n)^{-1}$ singularities in the light-cone gauge. These were designed to ensure that the location of the poles in the k^0-plane would not hinder Wick rotation nor spoil power-counting. These prescriptions have been tested successfully and they have allowed to handle those singular factors ensuing in the light-cone gauge [23–25]. Later on, it was shown [26,27] that the PV prescription alone does not and cannot work in dealing with light-cone poles because it violates causality. This causality can be recovered in the PV prescription if we consider the proper contribution of the so-called zero mode term, $\delta(k^+)$, and any prescription to treat those "spurious" poles that guarantees causality reproduce the same results as the ones obtained via the Mandelstam–Leibbrandt prescription.

The light-cone gauge $A^a_- = 0$ was also used to quantize massless Yang–Mills and derive the corresponding Feynman rules [28]. Following Dirac's procedure [9], the complete constraint structure and the generalized Dirac brackets for the physical variables were calculated in this gauge condition too [29].

In this paper we are going to show that if the null-plane gauge conditions [29] are used ghost fields decouple and only physical degrees of freedom left for the gauge potentials. This work is organized as follows. In Sect. 2, the transition amplitude to the electromagnetic field on the front-form is calculated using the null-plane gauge condition. In Sect. 3, the Hamiltonian functional generator for the Yang–Mills is given, where the appropriate integration measure is defined. The integration over the momenta determine the transition amplitude of the theory. Section 4 contains some further comments and remarks.

2 Electromagnetic Field

The starting point is the Lagrangian density of the free radiation field

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}. \tag{1}$$

Now, we can follow the steps of [6–8] to analyse the constraints *a la* Dirac [9] and find that there are two of the first-class type, i.e.,

$$\begin{aligned} \Psi_1 &\equiv \pi^+ \approx 0, \\ \Psi_2 &\equiv \partial_- \pi^- + \partial_i \pi^i \approx 0, \end{aligned} \tag{2}$$

and a set of second-class constraints

$$\phi^k \equiv \pi^k - \partial_- A_k + \partial_k A_- \approx 0. \tag{3}$$

As usual, the symbol in "≈ 0" means weak equality. Following Dirac's procedure, we choose two gauge conditions

$$\begin{aligned} \Delta_1 &\equiv A_- \approx 0 \\ \Delta_2 &\equiv \pi^- + \partial^x_- A_+ \approx 0; \end{aligned} \tag{4}$$